CSIR-UGC-NET/JRF June 2020

ELECTROMAGNETIC THEORY

ASSIGNMENT-1 : ELECTRIC FORCE, FIELD, FLUX, GAUSS LAW

OBJECTIVE QUESTIONS

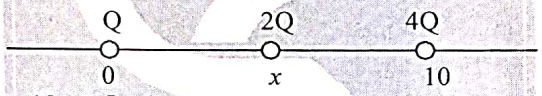
1. Consider two point charges q and λq located at the points, $x = a$ and $x = \mu a$, respectively. Assuming that the sum of the two charges is constant, what is the value of λ for which the magnitude of the electrostatic force is maximum? **[JEST 2015]**

(a) μ (b) 1 (c) $1/\mu$ (d) $1+\mu$

2. Two points charges $+q_1$ and $+q_2$ are fixed with a finite a distance 'd' between them. It is desired to put a third charge q_3 in between these two charges on the line joining them so that the charge q_3 is in equilibrium. That is:

(a) Possible only if q_3 is positive (b) Possible only if q_3 is negative
(c) Possible irrespective of the sign of q_3. (d) Not possible at all.

3. Three positively charged particles lie on a straight line at positions 0, x and 10 as indicated in the figure below. Their charges are Q, 2Q and 4Q cm respectively.



If the charges at x = 0 and x = 10 are fixed and the charge at x is movable, the system will be in equilibvrium when x =

(a) 8 (b) 2 (c) 20/3 (d) 10/3

4. Two point charges $Q_1 = 1$ nC and $Q_2 = 2$ nC are kept in free space such that the distance between them is 0.1 m. **[GATE 2001]**

(a) The force on Q_2 is along the direction from Q_2 to Q_1
(b) The force on Q_2 is the same in magnitude as that on Q_1
(c) The force on Q_1 is attractive
(d) A point charge $Q_3 = -3$ nC, placed at the midpoint between Q_1 and Q_2, experiences no net force.

5. When a particle of charge q is at rest, it produces

(a) Only electric field (b) Only magnetic field
(c) Both electric and magnetic fields. (d) A magnetic field depending upon the sign of the charge.

6. An electric field in a region is given by $\vec{E}(x, y, z) = ax\hat{i} + cz\hat{j} + 6by\hat{k}$. For which of a, b, c does this represent an electrostatic field? **[JEST 2012]**

(a) 13, 1, 12 (b) 17, 6, 1 (c) 13, 1, 6 (d) 45, 6, 1

7. If $\vec{E}_1 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$ and $\vec{E}_2 = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$ then **[JEST 2013]**

(a) Both are impossible electrostatic fields. (b) Both are possible electrostatic fields.
(c) Only $\vec{E}_1$ is a possible electrostatic field. (d) Only $\vec{E}_2$ is a possible electrostatic field.

8. Which of the following force fields is/are conservative ?

I. $(4x^3y + 3x^2z^2)\hat{i} + (x^4 + 3y^2z^2)\hat{j} + 2z(x^3 + y^3)\hat{k}$

II. $(yz^4 + x^5)\hat{i} + (x^4z + x^2y^3)\hat{j} + (x^4y + z^5)\hat{k}$

III. $x(3xy^2 + 4z^2)\hat{i} + (z^4 + 2x^3y)\hat{j} + 2z^2(2yz + 3x^2)\hat{k}$

(a) II and III only (b) I, II and III (c) I and III only
(d) I and II only (e) I only

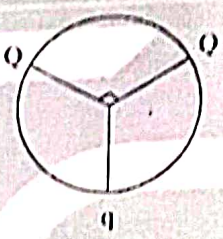
9. Which of the following vector fields can represent an electrostatic field?

(a) $\left(2xz - y^2\right)\hat{i} + \left(2xy - z^2\right)\hat{j} + \left(2yz - x^2\right)\hat{k}$

(b) $xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$ (c) $yz\hat{i} - xz\hat{j} + xy\hat{k}$

(d) $y^2\hat{i} + \left(2xy + z^2\right)\hat{j} + \left(2yz + x^2\right)\hat{k}$ (e) $y^2\hat{i} + \left(2xy + z^2\right)\hat{j} + 2yz\hat{k}$

10. Three charges are located on the circumference of a circle of radius 'R' as shown in the figure below. The two charges Q subtend an angle 90° at the centre of the circle. The charge 'q' is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q?



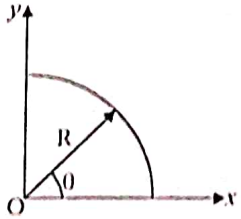
[NET Dec. 2012]

(a) $q/\sqrt{2}$ (b) $\sqrt{2}q$ (c) $2q$ (d) $4q$

11. A charge distribution has the charge density given by $\rho = Q\{\delta(x - x_0) - \delta(x + x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$ [GATE 2013]

(a) $\frac{2Q\hat{x}}{9\pi \epsilon_0 x_0^2}$ (b) $\frac{Q\hat{x}}{4\pi \epsilon_0 x_0^3}$ (c) $\frac{Q\hat{x}}{4\pi \epsilon_0 x_0^2}$ (d) $\frac{Q\hat{x}}{16\pi \epsilon_0 x_0^2}$

12. A segment of a circular wire of radius R, extending from $\theta = 0$ to $\pi/2$, carries a constant linear charge density, λ. The electric field at origin 'O' is:

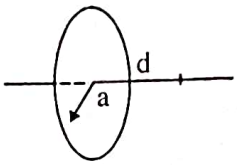


(a) $\frac{\lambda}{4\pi\varepsilon_0 R}(-\hat{x} - \hat{y})$ (b) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{\sqrt{2}}\hat{x} - \frac{1}{\sqrt{2}}\hat{y}\right)$ (c) $\frac{\lambda}{4\pi\varepsilon_0 R}\left(-\frac{1}{2}\hat{x} - \frac{1}{2}\hat{y}\right)$ (d) 0

13. Consider the plane defined by $(x, y, z = 10\ \text{m})$. If it carries a charge of $20\ \text{nC/m}^2$, what is the electric field intensity at the origin?

(a) $-720\pi\hat{k}\ \frac{\text{V}}{\text{m}}$ (b) $-360\pi\hat{k}\ \frac{\text{V}}{\text{m}}$ (c) $-72\pi\hat{k}\ \frac{\text{V}}{\text{m}}$ (d) $-18\ \pi\hat{k}\ \frac{\text{V}}{\text{m}}$

14. A very circular wire of radius a carries electric charge of uniform liner density ρ. On its axis, the magnitude of the electric field attains its maximum value at a perpendicular distance d form the plane of the wire (see the figure below). The value of d is



(a) 0 (b) 0.32 a (c) 0.50a (d) $a/\sqrt{2}$

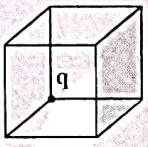
15. A circular loop of radius R, carries a uniform line charge density λ. The electric field, calculated at a distance z directly above the center of the loop, is maximum if z is equal to, **[JEST 2015]**

(a) $\frac{R}{\sqrt{3}}$ (b) $\frac{R}{\sqrt{2}}$ (c) $\frac{R}{2}$ (d) $2R$

16. A particle of mass m and charge q is constrained to move along a straight line joining two other equal charges q fixed at $x = \pm a$. The time period of small oscillation is:

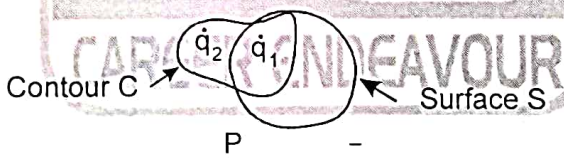
(a) $T = \frac{2\pi a}{q}\sqrt{\varepsilon_0 am}$ (b) $T = \frac{4\pi a}{q}\sqrt{\pi\varepsilon_0 am}$ (c) $T = \frac{a}{q}\sqrt{\pi\varepsilon_0 am}$ (d) $T = \frac{2\pi a}{q}\sqrt{\pi\varepsilon_0 am}$

17. A point charge 'q' sits at a corner of a cube of side 'a', as shown in the figure on the right. The flux of the electric field vector through the shaded side is **[TIFR 2013]**



(a) $\frac{q}{8\varepsilon_0}$ (b) $\frac{q}{16\varepsilon_0}$ (c) $\frac{q}{24\varepsilon_0}$ (d) $\frac{q}{6\varepsilon_0}$

18. Consider a set of two stationary point charges q_1 and q_2 as shown in the figure. Which of the following statements is correct? **[GATE 2002]**



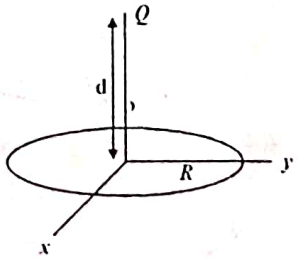
(a) The electric field at P is independent of q_2

(b) The electric flux crossing the closed surface S is independent of q_2

(c) The line integral of the electric field $\vec{E}$ over the closed contour C depends on q_1 and q_2.

(d) $\vec{\nabla}.\vec{E} = 0$ everywhere

19. The flux of electric field through a circle of radius R placed in the x-y plane with its centre at the origin due to a point charge Q placed at (0, 0, d) is

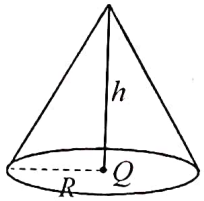


(a) $\frac{Q}{2\varepsilon_0}\left[1-\frac{d}{\left(d^2+R^2\right)^{1/2}}\right]$ (b) $\frac{Q}{2\varepsilon_0}\frac{d^3}{\left(d^2+R^2\right)^{3/2}}$ (c) $\frac{Q}{4\pi\varepsilon_0}\frac{d}{R}$ (d) $\frac{Q}{4\varepsilon_0}\frac{R^2}{d^2}$

20. An electron field $\vec{E}(\vec{r})=\frac{1}{r}\left(a\hat{r}+\beta\sin\theta\cos\phi\hat{\phi}\right)$ exists in space. What will be the total charge enclosed in a sphere of unit radius centered at the origin?

(a) $4\pi\varepsilon_0 a$ (b) $4\pi\varepsilon_0(\alpha+\beta)$ (c) $4\pi\varepsilon_0(\alpha-\beta)$ (d) $4\pi\varepsilon_0\beta$

21. Consider a charge Q at the origin of 3-dimensional coordinate system. The flux of the electric field through the curved surface of a cone that has a height 'h' and a circular base of radius R (as shown in the figure) is



(a) $\frac{Q}{\epsilon_0}$ (b) $\frac{Q}{2\epsilon_0}$ (c) $\frac{hQ}{R\epsilon_0}$ (d) $\frac{QR}{2h\epsilon_0}$ **[NET Dec. 2015]**

22. The potential due to an electric dipole p placed at the origin is known to be of the form $\phi(r)=p.r/r^3$. The total flux of the electric field through a spherical surface of radius R with the dipole at the centre is given by

(a) Zero (b) $-\frac{2\pi}{3}\frac{|p|}{R}$ (c) $+\frac{2\pi}{3}\frac{|p|}{R}$ (d) $+\frac{4\pi}{3}\frac{|p|}{R}$

23. The total outward electric flux going from a cube with $0\le x\le 1,\ 0\le y\le 1,\ 0\le z\le 1$ metres containing a volume charge density $\rho=16\,xyz$, will be

(a) 16 (b) 8 (c) 4 (d) 2

24. The charge density as a function of the radial distance r is given by **[JEST 2018]**

$$\rho(r)=\rho_0\frac{R^2-r^2}{R^2}$$

for $r<R$ and zero otherwise. The electric flux over the surface of an ellipsoid with axes $3R$, $4R$ and $5R$ centred at the origin is

(a) $\frac{4}{3\epsilon_0}\pi\rho_0R^3$ (b) $\frac{8}{9\epsilon_0}\pi\rho_0R^3$ (c) $\frac{8}{15\epsilon_0}\pi\rho_0R^3$ (d) 0

25. A non-conducting thin ellipsodial shell defined by the equation $x^2+2y^2+3z^2=a^2$ has a net chareg Q spread uniformly over its surface. The flux passing through a hemispherical surface defined by $x^2+y^2+z^2=a^2$ and $z>0$, is **[NET Dec. 2019]**

(a) $Q/\left(\sqrt{3}\,\epsilon_0\right)$ (b) Q/ϵ_0 (c) $Q/(2\epsilon_0)$ (d) $Q/(3\epsilon_0)$

26. Consider a hollow charged shell of inner radius 'a' and outer radius 'b'. The volume charge density is $\rho(r)=\frac{k}{r^2}$ (where k is a constant) in the region $a<r<b$. The magnitude of the electric field produced at distance $r>a$ is: **[NET Dec. 2012]**

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(a) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > a$ (b) $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{kb}{\varepsilon_0 r^2}$ for $r > b$

(c) $\frac{k(r-a)}{\varepsilon_0 r^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 r^2}$ for $r > b$ (d) $\frac{k(r-a)}{\varepsilon_0 a^2}$ for $a < r < b$ and $\frac{k(b-a)}{\varepsilon_0 a^2}$ for $r > b$

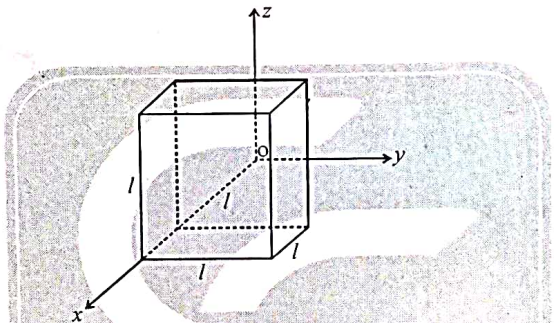
27. An electric charge distribution produces an electric field **[JEST 2018]**

$$\vec{E} = \left(1 - e^{-\alpha r}\right)\frac{\vec{r}}{r^3}$$

where δ and α are constants. The net charge within a sphere of radius α^{-1} centred at the origin is

(a) $4\pi \in_0 \left(1 - e^{-1}\right)$ (b) $4\pi \in_0 \left(1 + e^{-1}\right)$ (c) $-4\pi \in_0 \frac{1}{\alpha e}$ (d) $4\pi \in_0 \frac{1}{\alpha e}$

28. For an electric field $\vec{E} = k\sqrt{x}\hat{x}$, where k is a non-zero constant, total charge enclosed by the cube as shown below is **[JEST 2017]**

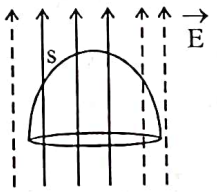


(a) 0 (b) $k\varepsilon_0 l^{5/2}\left(\sqrt{3} - 1\right)$ (c) $k\varepsilon_0 l^{5/2}\left(\sqrt{5} - 1\right)$ (d) $k\varepsilon_0 l^{5/2}\left(\sqrt{2} - 1\right)$

29. A static charge distribution gives rise to an electric field of the form $\vec{E} = \alpha\left(1 - e^{-r/R}\right)\frac{\hat{r}}{r^2}$, where α and R are positive constants. The charge contained within a sphere of radius R, centred at the origin is:

(a) $\pi\alpha\varepsilon_0 \frac{e}{R^2}$ (b) $\pi\alpha\varepsilon_0 \frac{e^2}{R^2}$ (c) $4\pi\alpha\varepsilon_0 \frac{R}{e}$ (d) $\pi\alpha\varepsilon_0 \frac{R^2}{e}$ **[NET Dec. 2011]**

30. A hemispherical surface of radius R is in a constant electric field $\vec{E}$ directed perpendicular to the base, as shown. The flux through the curved surface S of the hemisphere is

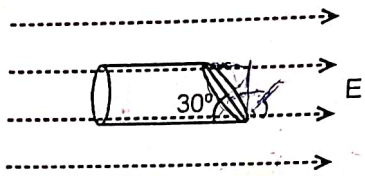


(a) 0 (b) $\pi R^2 E$ (c) $2\pi R^2 E$ (d) $4\pi R^2 E$

31. An electric field $\vec{E}(\vec{r}) = \frac{1}{r}\left(\alpha\hat{r} + \beta\sin\theta\cos\phi\hat{\phi}\right)$ exists in space. What will be the total charge enclosed in a sphere of unit radius centered at the origin?

(a) $4\pi\varepsilon_0\alpha$ (b) $4\pi\varepsilon_0(\alpha + \beta)$ (c) $4\pi\varepsilon_0(\alpha - \beta)$ (d) $4\pi\varepsilon_0\beta$

32. A cylindrical of radius R with the right side cut at an angle 30° as shown in the figure below is kept in a uniform electric field $\vec{E}$, with its axis along the direction of electric field.



The total electric flux linked with the entire cylinder is

(a) 0 (b) $-\left(\pi R^2 E\sqrt{3}\right)/2$ (c) $-\pi R^2 E/2$ (d) $\pi R^2 E$

33. Given a spherically symmetric charge density $\rho(r) = \begin{cases} kr^2 & ; \quad r < R \\ 0 & ; \quad r > R \end{cases}$, ($k$ being a constant), the electric field for $r < R$ is (take the total charge as Q)

(a) $\frac{Qr^3}{4\pi\varepsilon_0 R^5}\hat{r}$ (b) $\frac{3Qr^2}{4\pi\varepsilon_0 R^4}\hat{r}$ (c) $\frac{5Qr^3}{8\pi\varepsilon_0 R^5}\hat{r}$ (d) $\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$

34. A solid sphere of radius R has a charge density, given by $\rho(r) = \rho_0\left(1 - \frac{ar}{R}\right)$, where r is the radial coordinate and ρ_0, a and R are positive constants. If the magnitude of the electric field at $r = R/2$ is 1.25 times that at $r = R$, then the value of a is **[NET June 2014]**

(a) 2 (b) 1 (c) 1/2 (d) 1/4

35. A solid sphere of radius R carries a uniform volume charge density ρ. The magnitude of electric field inside the sphere at a distance r from the centre is : **[GATE 2005]**

(a) $\frac{r\rho}{3\varepsilon_0}$ (b) $\frac{R\rho}{3\varepsilon_0}$ (c) $\frac{R^2\rho}{r\varepsilon_0}$ (d) $\frac{R^3\rho}{r^2\varepsilon_0}$

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36. A sphere of inner radius 1 cm and outer radius 2 cm, centered at origins has a volume charge density $\rho_0 = \frac{K}{4\pi r}$, where K is a nonzero constant and r is the radial distance. A point charge of magnitude 10^{-3} C is placed at the origin. For what value of K in units of C/m², the electric field inside the shell is constant ? **[JEST 2017]**

37. A shpere of radius R carrries a volume charge density $\rho = \rho_0 r$, where ρ_0 is a constant and r is the radial distance from the centre of the charged sphere. The intensity of the electric field is proportional to:

(a) r^2 for $r \le R$ and $1/r^2$ for $r \ge R$ (b) r for $r \le R$ and $1/r^2$ for $r \ge R$

(c) $1/r^2$ for both $r \le R$ and $r \ge R$ (d) $1/r$ for $r \le R$ and $1/r^2$ for $r \ge R$

38. The magnitude of the electrostatic field of an infinite straight wire carrying a line charge density λ at a distance Z from it is given by

(a) $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda}{Z^2}$ (b) $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{Z^2}$ (c) $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{2\lambda}{Z}$ (d) $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{\lambda}{Z}$

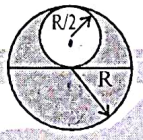
39. If $\vec{E} = \varepsilon_0\left[(2y^2+z)\hat{i} + 4xy\hat{j} + x\hat{k}\right]$ units of N/C the total charge enclosed by the cube $0 \le x < 1, 0 \le y < 1, \ 0 \le z \le 1$

(a) $2\varepsilon_0 C$ (b) $4\varepsilon_0 C$ (c) 0 (d) $\varepsilon_0 C$

40. The electric fields of a long straight wire varies with distance r (perpendicular to the wire) as

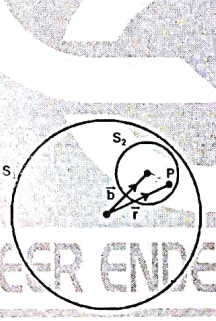
(a) $\frac{1}{r}$ (b) $\ln r$ (c) $1/r^2$ (d) $1/r^3$

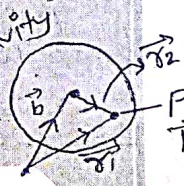
41. A sphere of radius R has a uniform charge density ρ. A sphere of smaller radius $\frac{R}{2}$ is cut out from the original sphere, as shown in the figure below. The centre of the cut out sphere lies at $z = \frac{R}{2}$. After the smaller sphere has been cut out, the magnitude of the electric field at $z = -\frac{R}{2}$ is $\frac{\rho R}{n\epsilon_0}$. The value of the integer n is 8 (12)



42. Consider a sphere S_1 of radius R which carries a uniform charge of density ρ. A smaller sphere S_2 of radius $a < \frac{R}{2}$ is cut out and removed from it. The centers of the two spheres are separated by the vector $\vec{b} = \hat{n}\frac{R}{2}$, as shown in the figure. **[NET June 2016]**

Calculate Potn at the centre of cavity. assume that sphere is centred at the origin (0,0,0)



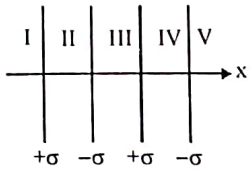


$\vec{b} + \vec{r}_2 = \vec{r}_1$

The electric field at a point P inside S_2 is

(a) $\frac{\rho R}{3\varepsilon_0}\hat{n}$ (b) $\frac{\rho R}{3\varepsilon_0 a}(\vec{r} - \hat{n}a)$ (c) $\frac{\rho R}{6\varepsilon_0}\hat{n}$ (d) $\frac{\rho a}{3\varepsilon_0 R}\vec{r}$

43. Four infinite conducting plates carry equal but opposite uniform charge densities $+\sigma$ as shown. The electrostatic fields in the regions I, II, III, IV and V are given respectively by



(a) $0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0$ (b) $\frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}, 0, \frac{\sigma}{\varepsilon_0}\hat{x}$ (c) $0, \frac{\sigma}{\varepsilon_0}\hat{x}, -\frac{\sigma}{\varepsilon_0}\hat{x}, \frac{\sigma}{\varepsilon_0}\hat{x}, 0$ (d) $0, +\frac{\sigma}{\varepsilon_0}\hat{x}, \frac{\sigma}{2\varepsilon_0}\hat{x}, +\frac{\sigma}{\varepsilon_0}\hat{x}, 0$

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44. Three infinite plane sheets carrying uniform charge densities $-\sigma, 2\sigma, 3\sigma$ are placed parallel to the xz-plane at $y = a, 3a, 4a$ respectively. The electric field at the point $(0, 2a, 0)$ is

(a) $\frac{4\sigma}{\varepsilon_0}\hat{j}$ (b) $-\frac{3\sigma}{\varepsilon_0}\hat{j}$ (c) $-\frac{2\sigma}{\varepsilon_0}\hat{j}$ (d) $\frac{\sigma}{\varepsilon_0}\hat{j}$

45. Two large nonconducting sheets one with a fixed uniform positive charge and another with a fixed uniform negative charge are placed at a distance of 1 meter from each other. The magnitude of the surface charge densities are $\sigma_+ = 6.8\mu C/m^2$ for the positively charged sheet and $\sigma_- = 4.3\mu C/m^2$ for the negatively charged sheet. What is the electric field in the region between the sheets? **[JEST 2014]**

(a) 6.30×10^5 N/C (b) 3.84×10^5 N/C (c) 1.40×10^5 N/C (d) 1.16×10^5 N/C

New 46. The charge per unit length of a circular wire of radius a in the xy-plane, with its center at the origin, is $\lambda = \lambda_0\cos\theta$, where λ_0 is a constant and the angle θ is measured from the positive x-axis. The electric field at the center of the circle is **[NET Dec. 2016]**

(a) $\vec{E} = -\frac{\lambda_0}{4\varepsilon_0 a}\hat{i}$ (b) $\vec{E} = \frac{\lambda_0}{4\varepsilon_0 a}\hat{i}$ (c) $\vec{E} = -\frac{\lambda_0}{4\varepsilon_0 a}\hat{j}$ (d) $\vec{E} = \frac{\lambda_0}{4\pi\varepsilon_0 a}\hat{k}$

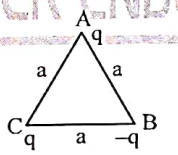
47. An infinitely long thin cylindrical shell has its axis coinciding with the z-axis. It carries a surface charge density $\sigma_0\cos\phi$, where ϕ is the polar angle and σ_0 is a constant. The magnitude of the electric field inside the cylinder is **[GATE 2019]**

(a) 0 (b) $\frac{\sigma_0}{2\varepsilon_0}$ (c) $\frac{\sigma_0}{3\varepsilon_0}$ (d) $\frac{\sigma_0}{4\varepsilon_0}$

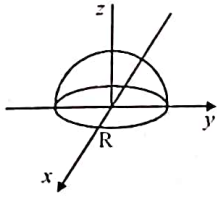
SUBJECTIVE QUESTIONS

1. A point charge $q = 50\mu C$ is located in the x-y plane at the point of the position vector $\vec{r}_0 = 2\hat{i} + 3\hat{j}$. What is the electric force on a point charge $q' = 20\mu C$ placed at point $\vec{r} = 8\hat{i} - 5\hat{j}$?

2. Three point charges are placed at the corners of an equilateral triangle of side 'a' as shown in figure. The Magnitude of force on charge at A is



3. A closed hemispherical bowl as shown in the figure, is placed in an electric field given by $\vec{E} = (r\cos\theta)\hat{r} + (r\sin\theta)\hat{\theta} + (r\sin\theta\cos\phi)\hat{\phi}$. Calculate the electric flux passing through the hemisphere.

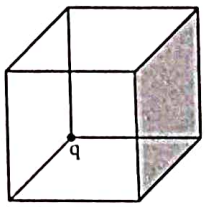


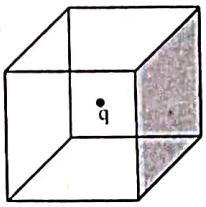
4. An electric field $\vec{E} = \alpha\hat{r} + f(\theta)\hat{\phi}$ exists in space. Total flux through a sphere centered at origin will

(a) depend on $f(\theta)$ (b) not dependent on radius of sphere

(c) depend both on $f(\theta)$ and radius (d) depend on radius but not on $f(\theta)$

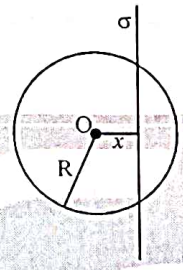
5. Calculate the flux of $\vec{E}$ through the shaded side?

(i) 

(ii) 

6. A cube of side a lies in the first octant with one corner at origin. An electric field $\vec{E} = E_0\left(1-\frac{x^2}{a^2}\right)\hat{i}$ exists in space. What is the total electric flux through the cube?

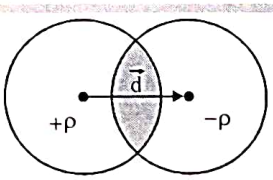
7. A plane sheet of surface charge density σ is inserted into a sphere of radius R. The perpendicular distance of the plane from the centre is x. Calculate the flux passing through the sphere.



8. If $\vec{E} = (2y^2 + z)\hat{x} + 4xy\hat{y} + x\hat{z}$ N/c, find
(a) The volume charge density at (–1, 0, 3)
(b) The flux through the cube defined by $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$
(c) The total charge enclosed by the cube

9. A long cylinder carries a charge density that is proportional to the distance from the axis: $\rho = ks$, for some constant k. Find the electric field inside this cylinder.

10. Imagine overlapping of two solid spheres of same radii and carrying uniform volume charges of densities ρ and $-\rho$. Calculate electric field in the overlapping region if distance between their centers is d.



11. Charge density on the surface of a hemisphere is $\sigma = \sigma_0 \cos\theta$, if the radius of the hemisphere is R then find the electric field at this centre of the hemisphere.

12. The electric field in a region is given to be $\vec{E} = ay\hat{i} + bx\hat{j} + cy\hat{k}$. Calculate the values of constants a, b and c, so that $\vec{E}$ is electrostatic field.

13. Find out which electric field is electrostatic?
(a) $\vec{E} = k[xy\hat{x} + 2yz\hat{y} + 3xz\hat{z}]$; (b) $\vec{E} = k[y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}]$

14. Electric field in a region is given to be $\vec{E} = \frac{1}{r}(\alpha\hat{r} + \beta \sin\theta \cos\phi\hat{\phi})$, calculate:

(i) The electric flux passing through a sphere of radius R centered at the origin.
(ii) Charge density ρ corresponding to this electric field.
(iii) Total charge enclosed by the sphere.

15. Determine which of the following are genuine electric fields (Electrostatic fields)

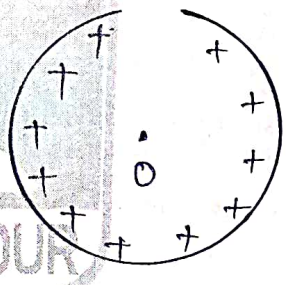
(a) $\vec{E}_1 = 5x^3\hat{x} + 15x^2y\hat{y}$

(b) $\vec{E}_2 = 2s(z+1)\sin\phi\hat{s} + s(z+1)\cos\phi\hat{\phi} + s^2\sin\phi\hat{z}$

(c) $\vec{E}_3 = \frac{5}{r}\sin\theta\cos\phi\,\hat{r}$

Ques.48
Jest 2020

A Ring of Radius 0.5m has a gap of 0.02π m. If the ring carries a charge of +1.0C distributed uniformly along it then the electric field at the centre of the ring is-



(a) $7.5\times10^7\,NC^{-1}$
(b) $7.2\times10^7\,NC^{-1}$
(c) $6.2\times10^7\,NC^{-1}$
(d) $6.5\times10^7\,NC^{-1}$

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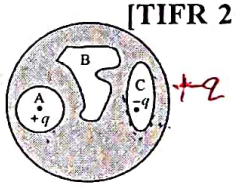
CSIR-UGC-NET/JRF June 2020

ELECTROMAGNETIC THEORY

ASSIGNMENT-2: ELECTRIC POTENTIAL AND CONDUCTORS

OBJECTIVE QUESTIONS

1. A solid spherical conductor encloses 3 cavities, a cross-section of which are as shown in the figure. A net charge $+q$ resides on the outer surface of the conductor. Cavities A and C contain point charges $+q$ and $-q$, respectively. **[TIFR 2015]**

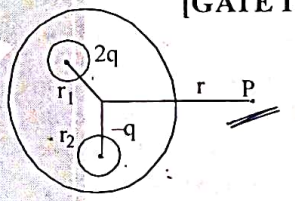


The net charges on the surfaces of these cavities are

(a) $A=-q, B=q, C=0$ (b) $A=-q, B=0, C=-q$

(c) $A=+q, B=0, C=-q$ (d) $A=-q, B=0, C=+q$

2. A spherical conductor of radius R has two cavities centered at r_1 and r_2 respectively from the centre of the conductor. The cavities contain point charges +2q and –q at their respective centers as shown in the figure. The magnitude of the electric field at a point P is : **[GATE 1996]**

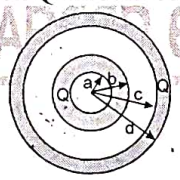


(a) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$ (b) $\frac{1}{4\pi\varepsilon_0}\left[\left(\frac{2q}{(r-r_1)^2}\right)^2+\left(\frac{-q}{(r-r_2)^2}\right)^2\right]^{1/2}$

(c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-r_1-r_2)^2}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q}{(r-r_1)^2+(r-r_2)^2}$

Common data for Q. 3 and Q.4.

Consider two concentric conducting spherical shells with inner and outer radii a, b and c, d as shown in the figure. Both the shells are given Q amount of positive charges.



3. The electric field in different regions are : **[GATE 2008]**

(a) $\vec{E}=0$ for $r<a$; $\vec{E}=\frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $a<r<b$; $\vec{E}=0$ for $b<r<c$; $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r>d$

(b) $\vec{E}=\frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r<a$; $\vec{E}=0$ for $a<r<b$; $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b<r<c$; $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r<d$

(c) $\vec{E}=\frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r<a$; $\vec{E}=0$ for $a<r<b$, $\vec{E}=0$ for $b<r<c$; $\vec{E}=\frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r>d$

(d) $\vec{E}=0$ for $r<a$; $\vec{E}=0$ for $a<r<b$; $\vec{E}=\frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b<r<c$; $\vec{E}=\frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r>d$

4. In order to have equal surface charge densities on the outer surfaces of both the shells, the following conditions should be satisfied **[GATE 2008]**

(a) $d = 4b$ and $c = 2a$ (b) $d = 2b$ and $c = \sqrt{2}a$

(c) $d = \sqrt{2}b$ and $c > a$ (d) $d > b$ and $c = \sqrt{2}a$

5. A hollow, closed conductor is placed in an external electric field $\vec{E}$. Then, at equilibrium, the electric field within the cavity is:

(a) $\vec{E}$ (b) 0

(c) Can be less than $\vec{E}$ but non-zero (d) Greater than $\vec{E}$

6. An electric charge, +Q is placed on the surface of a solid, conduction sphere of radius a. The distance measured from the centre of the sphere is denoted as r. Then : **[GATE 2003]**

(a) The charge gets distributed uniformly through the volume of the sphere
(b) The electrostatic potential has the same value for $r < a$
(c) An equal and opposite charge gets induced in the bottom half of the sphere
(d) The electric field is given by $1/\left(4\pi\varepsilon_0 r^2\right)$ for $r < a$

7. Equipotential surfaces corresponding to a particular charge distribution are given by $4x^2 + (y-2)^2 + z^2 = V_i$, where the values of V_i are constants. The electric field $\vec{E}$ at the origin is:

(a) $\vec{E} = 0$ (b) $\vec{E} = 2\hat{x}$ (c) $\vec{E} = 4\hat{y}$ (d) $\vec{E} = -4\hat{y}$

8. A shere of radius R has uniform volume charge density. The electric potential at a point r ($r < R$) is

(a) Due to the charge inside a sphere of radius r only **[GATE 2006]**
(b) Due to the entire charge of the sphere
(c) Due to charge in the spherical shell of inner and outer radii r and R, only
(d) Independent of r

9. Consider an infinite line charge with linear charge density λ. At a distance r from the line, the electrostatic potential has the form **[NET 2008]**

(a) $\frac{\lambda a}{4\pi\varepsilon_0 r}$ (b) $\frac{\lambda}{2\pi\varepsilon_0}\exp(-r/a)$ (c) $\frac{\lambda}{2\pi\varepsilon_0}\ln(\%)$ (d) $\frac{\lambda}{4\pi\varepsilon_0}\frac{r}{a}$

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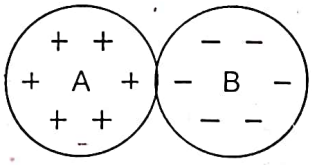
10. Two uniformly charged insulating solid spheres A and B, both of radius a, carry total charges $+Q$ and $-Q$, respectively. The spheres are placed touching each other as shown in the figure.



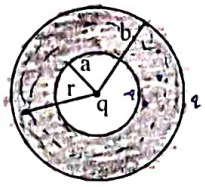
If the potential at the center of the sphere A is V_A and that at the center of B is V_B, then the difference $V_A - V_B$ is **[NET Dec. 2016]**

(a) $\frac{Q}{4\pi\varepsilon_0 a}$ (b) $\frac{-Q}{2\pi\varepsilon_0 a}$ (c) $\frac{Q}{2\pi\varepsilon_0 a}$ (d) $\frac{-Q}{4\pi\varepsilon_0 a}$

11. A loop of radius r carries a uniformly distributed charge. Assuming the potential at infinity to be zero, the ratio of the potential at a height '2r' on the axis to that at a height 3r is : **[IISc. 2008]**

(a) 1/2 (b) 2 (c) $\sqrt{1/2}$ (d) $\sqrt{2}$

12. A conducting spherical shell of inner radius a and outer radius b has a point charge q located at the centre of the shell. The potential at a distance r from the centre (a < r < b) is:



(a) 0 (b) $\frac{1}{4\pi\varepsilon_0}\frac{q}{a}$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{q}{b}$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

13. A uniformly charged thin spherical shell of radius R carries a total charge q. At a point inside (r < R) the spherical shell, if E and V are intensity of the electric field and electric potential respectively then : **[GATE 1998]**

(a) Both E and V are zero.
(b) E is zero but V is a nonzero constant.
(c) E is proportional to $1/r^2$ and V is proportional to $1/r$.
(d) Both E and V are nonzero constants.

14. Suppose that the electric field on the xy-plane is given by

$$\vec{E} = P\left[xy\hat{i} + \left(\frac{1}{2}x^2 + y^2\right)\hat{j}\right]$$

The magnitude of the difference in potential between the origin (0, 0) and the point (1, 2) is:

(a) $\frac{5}{4}P$ (b) $\frac{11}{3}P$ (c) $\frac{7}{2}P$
(d) Not unique, it depends on the path joining the two points.

15. Consider an infinitely long cylinder of radius R, placed along the z-axis, which carries a static charge density $\rho(r) = kr$, where r is the perpendicular distance from the axis of the cylinder and k is a constant. The electrostatic potential $\phi(r)$ inside the cylinder is proportional to **[TIFR 2015]**

(a) $-\frac{2}{3}\left(\frac{r^3}{R^3}+1\right)$ (b) $-2\ell n\left(\frac{r}{R}\right)$ (c) $-\frac{2}{3}\left(\frac{r^3}{R^3}-1\right)$ (d) $-2\ell n\left(\frac{R}{r}\right)$

16. Out of the following statements, choose the correct option(s) about a perfect conductor.
(a) The conductor has an equipotential surface
(b) Net charge, if any, resides only on the surface of conductor
(c) Electric field cannot exist inside the conductor
(d) Just outside the conductor, the electric field is always perpendicular to its surface

17. Identical charge q are placed at five vertices of a regular hexagon of side a. The magnitude of the electric field and the electrostatic potential at the centre of the hexagon are respectively **[GATE 2017]**

(a) 0, 0 (b) $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{q}{4\pi\varepsilon_0 a}$ (c) $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{5q}{4\pi\varepsilon_0 a}$ (d) $\frac{\sqrt{5}q}{4\pi\varepsilon_0 a^2}, \frac{\sqrt{5}q}{4\pi\varepsilon_0 a}$

18. The concentric thin spherical shells of radii a, b and c (a < b < c) carry uniform surface electric charge of densities $\sigma, -\sigma$ and σ, respectively. The electric potential at the surface of the outermost shell is

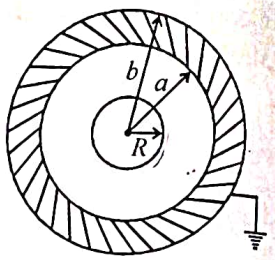
(a) $\frac{\sigma}{\varepsilon_0}(c-b+a)$ (b) $\frac{\sigma}{\varepsilon_0}\left(\frac{c^2}{a}-\frac{b^2}{a}+a\right)$ (c) $\frac{\sigma}{\varepsilon_0}\left(\frac{c^2}{b}-b\frac{a^2}{b}\right)$ (d) $\frac{\sigma}{\varepsilon_0}\left(c-\frac{b^2}{c}+\frac{a^2}{c}\right)$

19. A metal sphere of radius R with charge q is surrounded by a thick concentric spherical shell of inner radius a and outer radius b. The shell is grounded. The potential at the center of the sphere (with infinity as reference point) is given by

(a) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{b}+\frac{q}{R}-\frac{q}{a}\right)$ (b) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}\right)$

(c) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{b}-\frac{q}{R}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}-\frac{q}{a}\right)$

(e) $\frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R}+\frac{q}{a}\right)$



20. The electrostatic potential due to a charge distribution is given by $V(r)=A\frac{e^{-\lambda r}}{r}$, where A and λ are constants. The total charge enclosed within a sphere of radius $1/\lambda$, with its origin at $r=0$ is given by,

(a) $\frac{8\pi\varepsilon_0 A}{e}$ (b) $\frac{4\pi\varepsilon_0 A}{e}$ (c) $\frac{\pi\varepsilon_0 A}{e}$ (d) 0 **[JEST 2015]**

21. If the electrostatic potential in spherical polar coordinates is $\varphi(r)=\varphi_0 e^{-r/r_0}$ where φ_0 and r_0 are constants, then the charge density at a distance $r=r_0$ will be **[NET June 2014]**

(a) $\frac{\varepsilon_0\varphi_0}{er_0^2}$ (b) $\frac{\varepsilon\varepsilon_0\varphi_0}{2r_0^2}$ (c) $-\frac{\varepsilon_0\varphi_0}{er_0^2}$ (d) $-\frac{2\varepsilon\varepsilon_0\varphi_0}{r_0^2}$

22. The electrostatic potential in a given region of space is $\phi(\bar{r})=\frac{1}{\epsilon_0}\left[x^2+y^2-2z^2\right]$. The charge density $\rho(\bar{r})$ in the region is

(a) $\frac{-x-y+4z}{r^3}$ (b) $\frac{x+y-2z}{r}$ (c) zero (d) $\frac{4}{r^2}$

23. A solid, insulating sphere of radius 1 cm has charge 10^{-7} C distributed uniformly over its volume. It is surrounded concentrically by a conducting thick spherical shell of inner radius 2 cm, outer radius 2.5 cm and is charged with -2×10^{-7} C. What is the electrostatic potential in Volts on the surface of the sphere ? **[JEST 2017]**

24. If the electrostatic potential were given by $\phi=\phi_0\left(x^2+y^2+z^2\right)$, where ϕ_0 is constant, then the charge density giving rise to the above potential would be : **[GATE 2005]**

(a) 0 (b) $-6\,\phi_0\,\varepsilon_0$ (c) $-2\,\phi_0\,\varepsilon_0$ (d) $-\frac{6\phi_0}{\varepsilon_0}$

25. The electrostatic potential V(x,y) in free space in a region where the charge density ρ is zero is given by $V(x,y)=4e^{2x}+f(x)-3y^2$. Given that the x-component of the electric field, E_x, and V are zero at the origin, f(x) is: **[NET June 2011]**

(a) $3x^2-4e^{2x}+8x$ (b) $3x^2-4e^{2x}+16x$ (c) $4e^{2x}-8$ (d) $3x^2-4e^{2x}$

26. If the electrostatic potential $V(r,\theta,\phi)$ in a charge free region has the form $V(r,\theta,\phi)=f(r)\cos\theta$, then the functional form of $f(r)$ (in the following a and b are constants) is **[NET Dec. 2013]**

(a) $ar^2+\frac{b}{r}$ (b) $ar+\frac{b}{r^2}$ (c) $ar+\frac{b}{r}$ (d) $a\ln\left(\frac{r}{b}\right)$

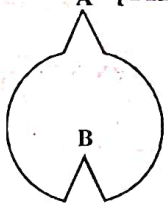
27. The one dimensional potential $V(x)$ for an electrostatic fields in a region of space which does not contain any charge has the form (in the following a and b are constants)

(a) $a\sin bx$ (b) ae^{-x^2} (c) $\frac{1}{2}ax^2$ (d) $ax+b$

28. Consider two concentric spherical metal shells of radii r_1 and r_2 $(r_2 > r_1)$. The outer shell has a charge q and the inner shell is grounded. What is the charge on the inner shell? **[JEST 2019]**

(a) $-\frac{r_1}{r_2}q$ (b) $\frac{r_1}{r_2}q$ (c) 0 (d) $\frac{r_2}{r_1}q$

29. A solid spherical conductor has a conical hole made at one end, ending in a point B, and a small conical projection of the same shape and size at the opposite side, ending in a point A. A cross-section through the centre of the conductor is shown in the figure on the right. **[TIFR 2014]**



If now, a positive charge Q is transferred to the sphere, then
(a) The charge density at both A and B will be undefined.
(b) The charge density at A will be the same as the charge density at B.
(c) The charge density at A will be more than the charge density at B.
(d) The charge density at B will be more than the charge density at A.

30. Two metallic sphere of radius 1 cm and 3 cm respectively are separated by a large distance D, $(D \gg 1\text{cm})$. Initially, the smaller sphere carries a charge Q while the larger one is uncharged. If a thin metallic wire is connected between the two spheres, the ratio of the charges on the smaller sphere to the larger on in equilibrium will be : **[IISc. 2012]**

(a) 1/3 (b) 1/9 (c) 3 (d) 9

31. A charge $+q$ is brought near an isolated metal cube having no charge initially
(a) the cube becomes positively charged
(b) the cube becomes negatively charged
(c) the external surface becomes negatively charged and the interior becomes positively charged
(d) the interior remains charge free and the surface gets non-uniform charge distribution

32. For a scalar function φ satisfying the Laplace equation, $\nabla\varphi$ has **[GATE 2013]**

(a) zero curl and non-zero divergence (b) non-zero curl and zero divergence
(c) zero curl and zero divergence (d) non-zero curl and non-zero divergence

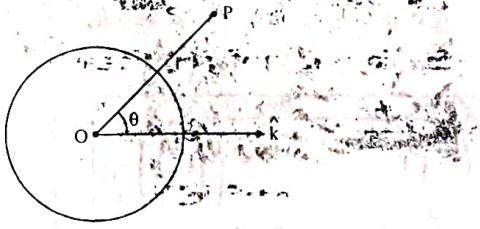
33. A conducting sphere of radius R is placed in a uniform electric field $\vec{E}_0$ directed along + z axis. The electric potential for outside points is given as : **[GATE 2004]**

$V_{out} = -E_0\left(1-\frac{R^3}{r^3}\right)r\cos\theta$, where r is the distance from the centrer and θ is the polar angle. The charge density on the surfae of the sphere is :

(a) $3\varepsilon_0 E_0\cos\theta$ (b) $\varepsilon_0 E_0\cos\theta$ (c) $3\varepsilon_0 E_0\cos\theta$ (d) $\frac{\varepsilon_0}{3}E_0\cos\theta$

34. A spherical conductor of radius a is placed in a uniform electric field $\vec{E} = E_0\hat{k}$. The potential at a point $P(r,\theta)$ for r > a, is given by, $\varphi(r,\theta) = \text{constant} - E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$ **[GATE 2011]**

Where r is the distance of P from the centre O of the sphere and θ is the angle OP makes with the z-axis.



The charge density on the sphere at $\theta = 30°$ *is* :

(a) $3\sqrt{3}\,\varepsilon_0 E_0/2$ (b) $3\varepsilon_0 E_0/2$ (c) $\sqrt{3}\varepsilon_0 E_0/2$ (d) $\varepsilon_0 E_0/2$

35. The yz-plane at $x = 0$ carries a uniform surface charge density σ. A unit point charge is moved from a point $(\delta, 0, 0)$ on one side of the plane to a point $(-\delta, 0, 0)$ on the other side. If δ is an infinitesimally small positive number, the work done in moving the charge is **[NET Dec. 2019]**

(a) 0 (b) $\frac{\sigma}{\varepsilon_0}\delta$ (c) $-\frac{\sigma}{\varepsilon_0}\delta$ (d) $\frac{2\sigma}{\varepsilon_0}\delta$

36. Electric charges Q and –Q are distributed uniformly on the surfaces of two concentric shells of radii a and b, respectively $(a < b)$. The energy required to assemble this charge distribution is

(a) $\frac{Q^2}{4\pi\varepsilon_0}\frac{(b-a)}{(b+a)^2}$ (b) $\frac{Q^2}{8\pi\varepsilon_0}\frac{(b-a)}{ab}$ (c) $\frac{Q^2}{4\pi\varepsilon_0}\frac{(b-a)}{b^2}$ (d) $\frac{Q^2}{8\pi\varepsilon_0}\frac{(b-a)}{b^2}$

37. Three equal chrages Q are successively brought from infinity and each is placed at one of the three vertices of an equilateral tringle. Assuming the rest of the Universe as a whole to be neutral, the energy E_0 of the electrostatic field will increase, successively, to **[TIFR 2012]**

$E_0 + \Delta_1$, $E_0 + \Delta_1 + \Delta_2$, $E_0 + \Delta_1 + \Delta_2 + \Delta_3$ Where $\Delta_1 : \Delta_2 : \Delta_3 =$

(a) 1 : 2 : 3 : (b) 1 : 1: 1 (c) 0 : 1 : 1 (d) 0 : 1 : 2

38. The electric fields outside $(r > R)$ and inside $(r < R)$ a solid sphere with a uniform volume charge density are given by $\vec{E}_{r>R} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$ and $\vec{E}_{r<R} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^3}r\hat{r}$ respectively, while the electric field outside a spherical shell with a uniform surface charge density is given by $\vec{E}_{r>R} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$, q being the total charge. The correct ratio of the electrostatic energies for the second case to the first case is **[JEST 2013]**

(a) 1:3 (b) 9:16 (c) 3:8 (d) 5:6

39. A conducting sphere of radius R has charge + Q on its surface. If the charge on the sphere is doubled and its radius is halved, the energy associated with the electric field will : **[GATE 2004]**

(a) Increase four times (b) Increase eight times
(c) Remain the same (d) Decrease four times

40. The amount of energy required to charge a metallic sphere of radius 2.0 m to a potential of 3000 V in air is:

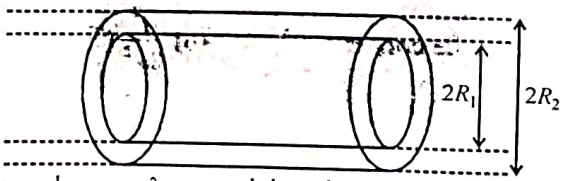
(a) 1×10^{-3} Joule (b) 2×10^{-3} Joule (c) 3×10^{-3} Joule (d) 4×10^{-3} Joule

41. The electrostatic energy (in units of $\frac{1}{4\pi\varepsilon_0}$ J) of a uniformly charged spherical shell of total charge 5C and radius 4 m is 3.125 (Round off to 3 decimal places)

42. A small charged spherical shell of radius 0.01 m is at a potential of 30V. The electrostatic energy of the shell is

(a) 10^{-10} J (b) 5×10^{-10} J (c) 5×10^{-9} J (d) 10^{-9} J

43. Two long hollow co-axial conducting cylinders of radii R_1 and R_2 $(R_1 < R_2)$ are placed in vacuum as shown in the figure below.

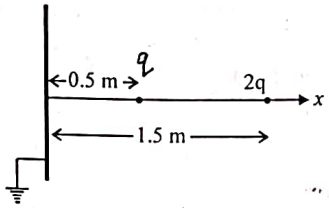


The inner cylinder carries a charge $+\lambda$ per unit length and the outer cylinder carries a charge $-\lambda$ per unit length. The electrostatic energy per unit length of this system is **[NET June 2017]**

(a) $\frac{\lambda^2}{\pi\varepsilon_0}\ln\left(\frac{R_1}{R_2}\right)$ (b) $\frac{\lambda^2}{4\pi\varepsilon_0}\left(\frac{R_2^2}{R_1^2}\right)$ (c) $\frac{\lambda^2}{4\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$ (d) $\frac{\lambda^2}{2\pi\varepsilon_0}\ln\left(\frac{R_2}{R_1}\right)$

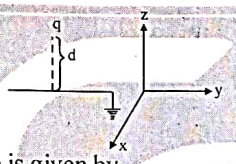
44. Calculate the self-energy, in Joules, of a spherical conductor of radius 8.5 cm, which carries a charge 100 μC. **[TIFR 2018]**

45. Two charges q and 2q are placed along the x-axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge q is: **[GATE 2010]**



(a) $\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$ (b) $\frac{1}{4\pi\varepsilon_0}2q^2$ (c) $\frac{1}{4\pi\varepsilon_0}q^2$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}$

46. Let a point charge +q be at a distance d from an infinite conducting plate which is grounded.



The force experienced by the charge is given by

(a) $\vec{F} = +\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}\hat{k}$ (b) $\vec{F} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{d^2}\hat{k}$ (c) $\vec{F} = -\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2}\hat{k}$ (d) $\vec{F} = +\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2}\hat{k}$

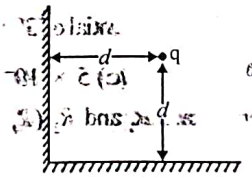
47. A point charge is placed between two semi-infinite conducting plates which are inclined at an angle of 30° with respect to each other. The number of image charges is **[GATE 2015]**

48. A point charge 'q' of mass 'm' is kept at a distance 'd' below a grounded infinite conducting sheet which lies in the xy-plane. What is the value of 'd' for which the charge remains stationary?

(a) $q/4\sqrt{mg\pi\varepsilon_0}$ (b) $q/\sqrt{mg\pi\varepsilon_0}$ **[NET Dec. 2012]**

(c) There is no finite value of 'd' (d) $\sqrt{mg\pi\varepsilon_0/q}$

49. A point charge q is placed symmetrically at a distance d from two perpendicularly placed grounded conducting infinite plates as shown in the figure. The net force on the charge (in units of $1/4\pi\epsilon_0$) is **[NET Dec. 2013]**



(a) $\frac{q^2}{8d^2}(2\sqrt{2}-1)$ away from the corner (b) $\frac{q^2}{8d^2}(2\sqrt{2}-1)$ towards the corner

(c) $\frac{q^2}{2\sqrt{2}d^2}$ towards the corner (d) $\frac{3q^2}{8d^2}$ away from the corner

50. A charge q is placed in front of an infinite metal plate at a distance d. If instead a charge 2q were to experience the same force, at what distance should it be placed from the plate? **[IISc. 2008]**

(a) d (b) 2d (c) d/2 (d) 4d

51. A charged particle is at a distance d from an infinite conducting plane maintained at zero potential. When released from rest, the particle reaches a speed u at a distance d/2 from the plane. At what distance from the plane will the particle reach the speed 2u? **[NET June 2014]**

(a) d/6 (b) d/3 (c) d/4 (d) d/5

52. A charge (–e) is placed in vacuum at the point (d, 0, 0), where d > 0. The region $x \le 0$ is filled uniformly with a metal. The electric field at the point $\left(\frac{d}{2}, 0, 0\right)$ is **[NET June 2014]**

(a) $-\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$ (b) $\frac{10e}{9\pi\varepsilon_0 d^2}(1,0,0)$ (c) $\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$ (d) $-\frac{e}{\pi\varepsilon_0 d^2}(1,0,0)$

53. A point charge +q placed at (0, 0, d) above a grounded infinite conducting plane defined by z = 0. There are no charges present anywhere else. What is the magnitude of electric field at (0, 0, –d) ?

(a) $q/(8\pi\varepsilon_0 d^2)$ (b) $-\infty$ (c) 0 (d) $q/(16\pi\varepsilon_0 d^2)$ **[JEST 2012]**

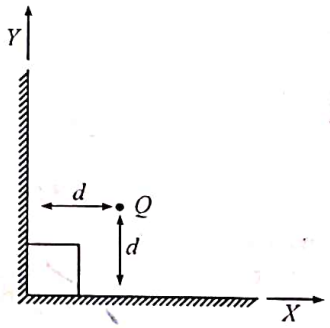
54. A point charge is at a distance d from a metal plate. The force acting on the charge is:

(a) Proportional to $1/d^2$ and repulsive. (b) zero
(c) Proportional to $1/d^2$ and attractive. (d) Proportional to 1/d and attractive.

55. A point charge q is kept at the mid-point between two large parallel grounded conducting plates. Assume no gravity. The charge is displaced a little towards the right plate. The charge will now, **[GATE 2000]**

(a) Stay where it is (b) Move towards the right plate
(c) Move towards the left plate (d) Oscillate between the plates

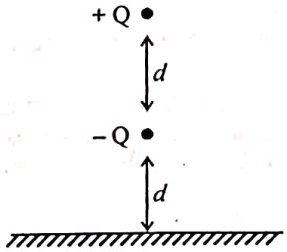
56. A positive point charge Q is located at a distance d from two grounded conducting half planes perpendicular to each other, as shown in the figure. The force on Q caused by the charges induced on the planes is



(a) 0 (b) $\frac{Q^2}{4\pi\epsilon_0 d^2}\left(\frac{-1}{4}\hat{i} - \frac{1}{4}\hat{j}\right)$

(c) $\frac{Q^2}{4\pi\epsilon_0 d^2}\left(\frac{-1}{2\sqrt{2}}\hat{i} - \frac{1}{2\sqrt{2}}\hat{j}\right)$ (d) $\frac{Q^2}{4\pi\epsilon_0 d^2}\left(\frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{i} + \frac{1-2\sqrt{2}}{8\sqrt{2}}\hat{j}\right)$

57. Two points charges +Q and –Q are placed away from a grounded conducting plane as shown in the figure. The net force on the charge –Q is



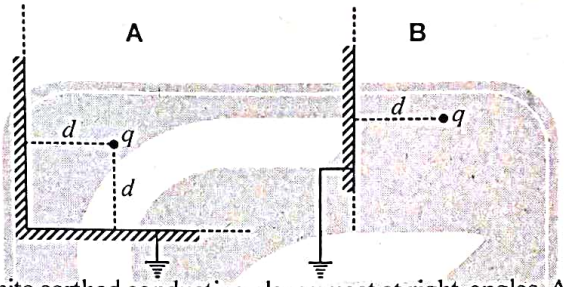
(a) $\frac{Q^2}{4\pi \epsilon_0 d^2}$ away from the conducting plane

(b) $\frac{Q^2}{4\pi \epsilon_0 d^2}\left(\frac{137}{144}\right)$ towards the conducting plane

(c) $\frac{Q^2}{4\pi \epsilon_0 d^2}\left(\frac{31}{36}\right)$ away from the conducting plane

(d) $\frac{Q^2}{4\pi \epsilon_0 d^2}\left(\frac{137}{144}\right)$ away from the conducting plane

58. Consider the following situations.



In situation A, two semi-infinite earthed conducting planes meet at right-angles. A point charge q, is placed at a distance d from each plane, as shown in the figure A. The magnitude of the force exerted on the charge q is denoted F_A.

In situation B, the same charge q is kept at the same distance d from an infinite earthed conducting plane, as shown in the figure B. The magnitude of the force exerted on the charge q is denoted F_B.

Find the numerical ratio F_A/F_B. **[TIFR 2017]**

59. Consider a gounded conducting plane which is infinitely extended perpendicular to the y-axis at $y = 0$. If an infinite line of charge per unit length λ runs parallel to x-axis at $y = d$, then surface charge density on the conducting plane is **[JEST 2017]**

(a) $\frac{-\lambda d}{(x^2 + d^2 + z^2)}$ (b) $\frac{-\lambda d}{(x^2 + d^2 + z^2)}$ (c) $\frac{-\lambda d}{\pi(x^2 + d^2 + z^2)}$ (d) $\frac{-\lambda d}{2\pi(x^2 + d^2 + z^2)}$

60. A point charge q of mass m is released from rest at a distance d from an infinite grounded conducting plane (ignore gravity). How long does it takes for the charge to hit the plane ?

(a) $\frac{\sqrt{2\pi^3 \varepsilon_0 m d^3}}{q}$ (b) $\frac{\sqrt{2\pi^3 \varepsilon_0 m d}}{q}$ (c) $\frac{\sqrt{\pi^3 \varepsilon_0 m d^3}}{q}$ (d) $\frac{\sqrt{\pi^3 \varepsilon_0 m d}}{q}$ **[JEST 2016]**

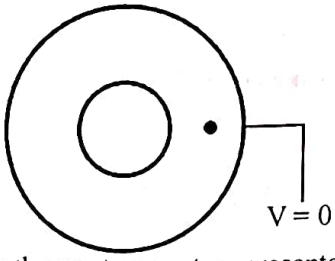
61. Two point charges $+3Q$ and $-Q$ are placed at $(0, 0, d)$ and $(0, 0, 2d)$ respectively, above an infinite grounded conducting sheet kept in the xy-plane. At a point $(0, 0, z)$, where $z \gg d$, the electrostatic potential of this charge configuration would approximately be **[NET Dec. 2017]**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{d^2}{z^3}Q$ (b) $\frac{1}{4\pi\varepsilon_0}\frac{2d}{z^2}Q$ (c) $\frac{1}{4\pi\varepsilon_0}\frac{3d}{z^2}Q$ (d) $-\frac{1}{4\pi\varepsilon_0}\frac{d^2}{z^3}Q$

62. A charge + q is kept at a distance of 2R from the center of a grounded conducting sphere of radius R. The image charge and its distance from the centre are, respectively : **[GATE 2004]**

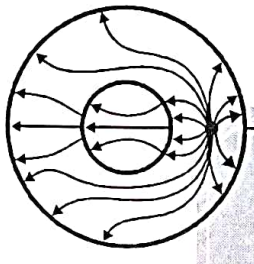
(a) $-\frac{q}{2}$ and $\frac{R}{2}$ (b) $-\frac{q}{2}$ and $\frac{R}{4}$ (c) $-q$ and $\frac{R}{2}$ (d) $+\frac{q}{2}$ and $\frac{R}{2}$

63. Two long hollow conducting cylinders, each of height h, are placed concentrically on the ground, as shown in the figure (top view). The outer cylinder is grounded, while the inner cylinder is insulated. A positive charge (the black dot in the figure) is placed between the cylinders at a height $h/2$ from the ground. **[TIFR 2017]**

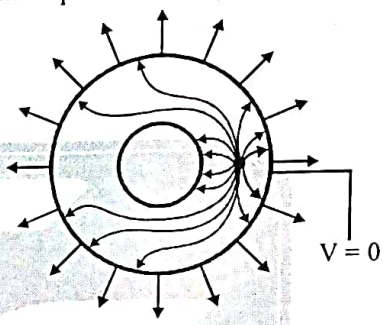


Which of the following figures gives the most accurate representation (top view) of the lines of force ?

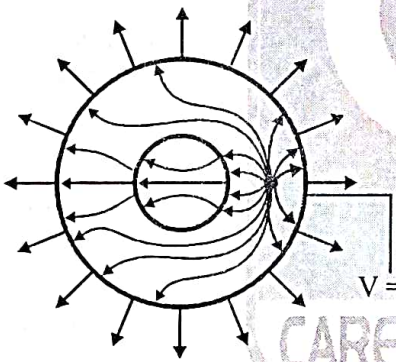
(a)



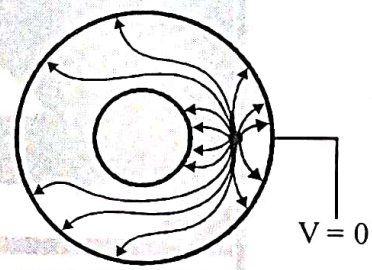
(b)



(c)



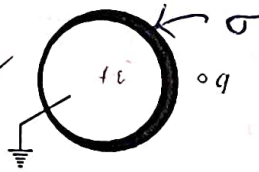
(d)



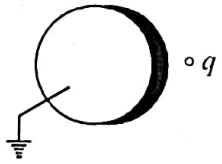
64. A point charge $q < 0$ is brought in front of a grounded conducting sphere. If the induced charge density on the sphere is plotted such that the thickness of the black shading is proportional to the charge density, the correct plot will most closely resemble **[TIFR 2019]**

(a)

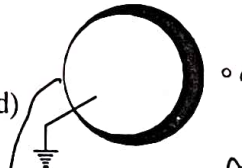
(b)



(c)



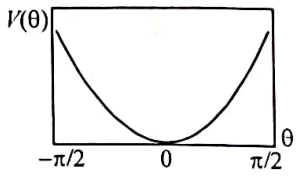
(d)

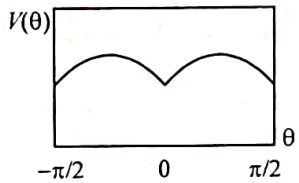


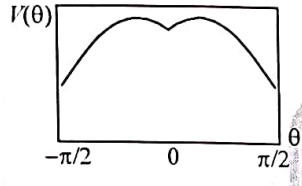
CAREER ENDEAVOUR

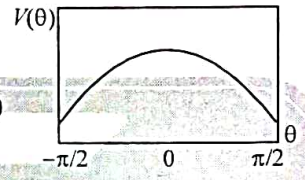
65. A point charge +q is placed outside a grounded conducting sphere of radius a. Which of the following statements is not true?
(a) there is an attractive force between the sphere and the point charge
(b) the induced surface charge density on the sphere is not the same everywhere
(c) the electric field inside the sphere is zero
(d) the total induced charge on the sphere is -q $\rightarrow -q\left(\frac{R}{a}\right)$

66. A solid metal cylinder, of radius R and height $4R$, is resting on its base on a horizontal table. When the cylinder is tilted on one of its edges, the variation of its potential energy $V(\theta)$, as a function of the tilt angle θ, is qualitativley best represented by wich of the following figures? **[NET Dec. 2019]**

(a) 

(b) 

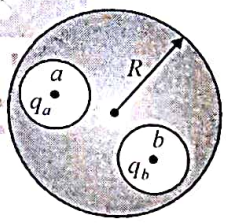
(c) 

(d) 

SUBJECTIVE QUESTIONS

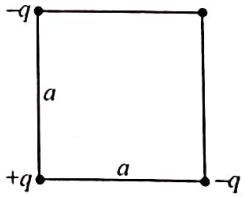
1. Two conducting spheres of charges Q_1 and Q_2 and radius r_1 and r_2 respectively, such that $(r_1 > r_2)$, now these two conductors are connected by a metallic wire. Calculate the potential and charge on two spheres after connection.

2. Two spherical cavities, of radii a and b, are hollowed out from the interior of a (neutral) conducting sphere of radius R shown in figure. At the centre of each cavity a point charge is placed - call these charges q_a and q_b.



(a) Find the surface charges σ_a, σ_b and σ_R.
(b) What is the field outside the conductor?
(c) What is the field within each cavity?
(d) What is the force on q_a and q_b?
(e) Which of these answers would change if a third charge, q_c, were brought near the conductor?

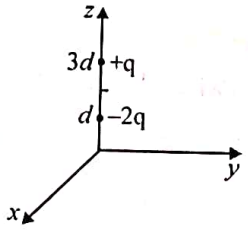
3. (a) Three charges are situated at the corners of a square (side a), as shown in figure. How much work does it take to bring in another charge, $+q$, from far away and place it in the fourth corner?
(b) How much work does it take to assemble the whole configuration of four charges?



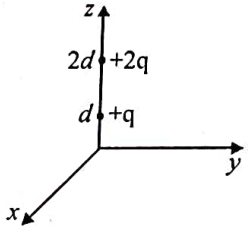
4. Find the energy of a uniformly charged spherical shell of total charge q and radius R.

5. Find the force on the charge $+q$ in figure. (The xy plane is a grounded conductor).

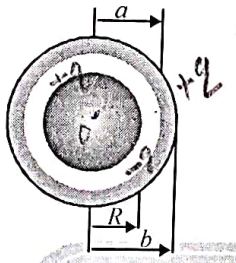
(a)



(b)



6. A metal sphere of radius R, carrying charge q, is surrounded by a thick concentric metal shell (inner radius a, outer radius b, as shown in figure). The shell carries no net charge.



(a) Find the surface charge density σ at R, at a, and at b.

(b) Find the potential at the center, using infinity as the reference point.

(c) Now the outer surface is touched to a grounding wire, which lowers its potential to zero (same as at infinity). How do your answers to (a) and (b) change?

7. A sphere of radius R carries a charge density $\rho(r) = kr$ (where k is a constant). Find the energy of this charge configuration.

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